→ Cardinal points of a thick lens & thick lens formula:

The cardinal points of an optical-system or a thick lens are six in number
(a) Two focal points.
(b) Two principal points.
(c) Two nodal points.

A pair of points lying on the principal axis of optical system and conjugate to points at infinity are called focal points. These are two as shown in the fig(i) & (iv)

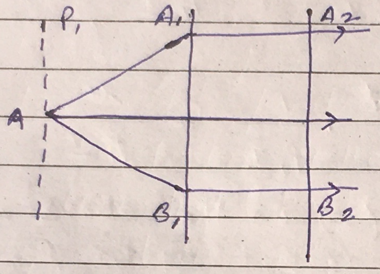


fig. (1)

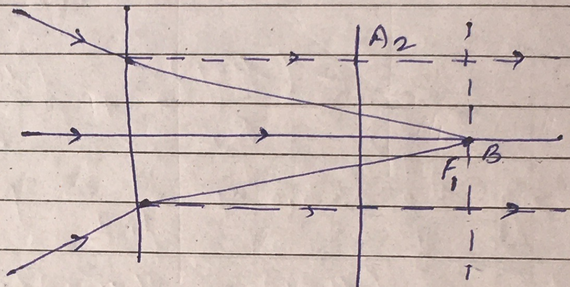


fig. (2)

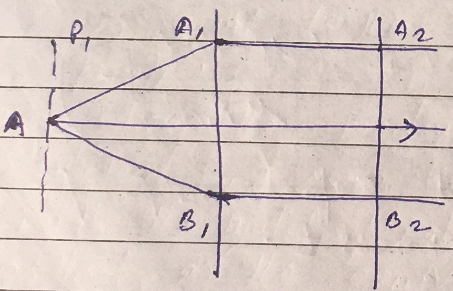


fig (3)

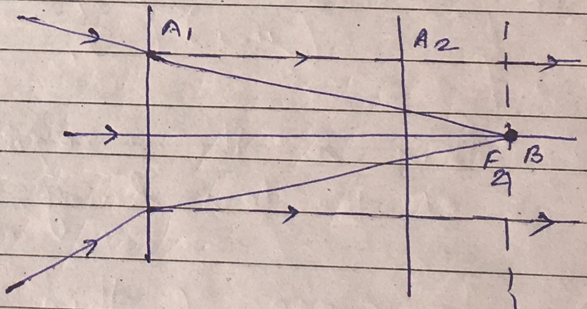
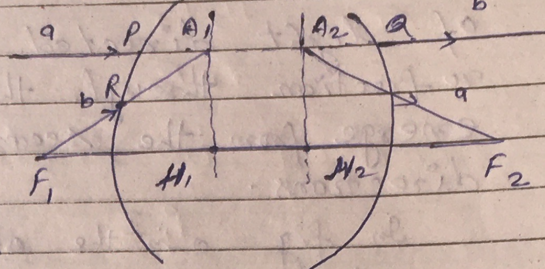


fig (4)

1. First Focal point & First focal plane :→ It is the object point on the principal axis for which image point is on the infinity. The plane passing through this and perpendi-cular

to the principal axis is focal plane.

The second focal point of an optical system is defined as the image point on the principal axis for which the object lies at infinity.



(2) Principal points :→ A pair of conjugate points on the principal axis of the optical system or a thick lens having unit positive linear magnification are called principal points of the optical system of a thick lens.

The points H_1 and H_2 shown in the above fig. are the principal points. It is obvious from this fig. that the incident rays 'a' and 'b' are converging towards at points A_1 and after refraction the corresponding emergent rays appear to diverge from point A_2. Hence A_2 is the image of A_1 where $AH_1 = AH_2$ and both these are ⊥r. to the principal axis at points H_1 & H_2 respectively. These two points are the principal points and A_1H_1 and A_2H_2 are principal planes.

The main features of these points in the lens are :

(i) If an incident ray passes through a point in the first principal plane at a given distance from the axis, the corresponding emergent ray will certainly pass through a point in the second plane at the same distance from the axis.

(ii) If the medium on the both the sides of the optical system is same, the first and second focal length of an optical system will be equal

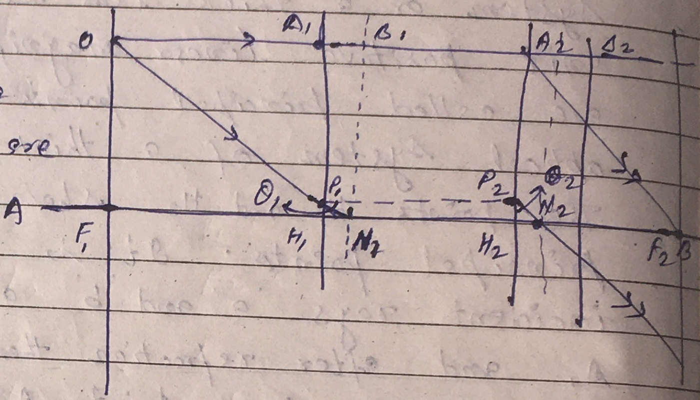
i.e. $f_1 = f_2$.

(11)

③ Nodal points :→ A pair of conjugate points on the principal axis of the optical system having unit positive angular magnification are called the nodal points of the optical system.

This simply means that a ray of light directed towards one of these points after refraction through the optical system, will appear to emerge from the second nodal point parallel to the original directions.

In fig 'O' is the object and 'I' is the corresponding image. The point N_1 & N_2 are two nodal points.



The feature of the two nodal points are

(i) $$\frac{\tan\theta_1}{\tan\theta_2} = 1$$

(ii) $H_1H_2 = N_1N_2$ i.e. distance between two nodal points is equal to the distance between two principal points.

(iii) When the medium on both the sides of the optical system is same (let air), the principal points coincides with the nodal points and then these points are then called as the equivalent points.

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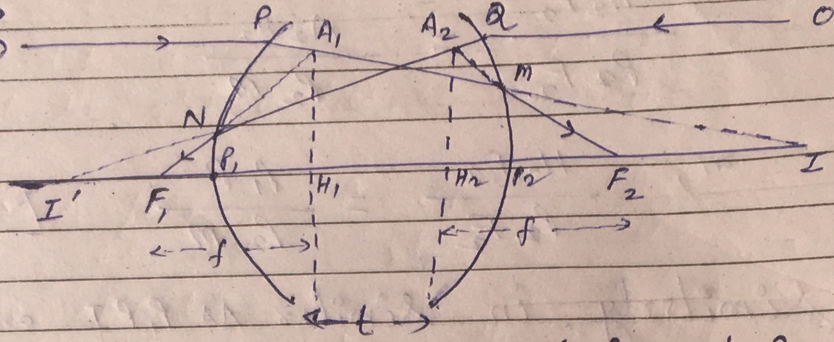
Thick lens formula: →

Focal length of a thick lens i.e. thickness 't' and radii of curvature R_1 & R_2 : →



Let us consider the above figure in which f_1 and f_2 be the first and second focal lengths of the lens respectively. 'μ' be the refractive index of the medium on the either side be air.

We have from figure

$$H_1F_1 = f_1 \quad \& \quad H_2F_2 = f_2$$

I be the position of the image formed by the first surface PP_1 when object is at infinity. Let $P_1I = v'$

we have $$\frac{\mu}{v'} - \frac{1}{\infty} = \frac{\mu - 1}{R_1}$$

or, $$\frac{\mu}{P_1I} = \frac{\mu - 1}{R_1} \quad \text{---} \quad (1)$$

or, $$\frac{1}{P_1I} = \frac{(\mu - 1)}{\mu R_1} \quad \text{---} \quad (2)$$

For refraction at second surface

$$\frac{1}{P_2F_2} - \frac{\mu}{P_2I} = \frac{1}{R_2} - \frac{\mu}{R_2}$$

or, $$\frac{1}{P_2F_2} - \frac{\mu}{P_2I} = \frac{1 - \mu}{R_2} \quad \text{---} \quad (3)$$

Now in similar triangles $A_2F_2H_2$ and MP_2F_2 :

$$\frac{H_2F_2}{P_2F_2} = \frac{A_2H_2}{P_2M}$$

or, $$\frac{H_2F_2}{P_2F_2} = \frac{P_1P}{P_2M} \quad [A_2H_2 = P_1P] \quad \text{—} (4)$$

Similarly in similar Δ's P_1PI and MP_2I

$$\frac{P_1I}{P_2I} = \frac{P_1P}{P_2M} \quad \text{—} (5)$$

From (4) & (5) we have

$$\frac{H_2F_2}{P_2F_2} = \frac{P_1I}{P_2I}$$

or, $$\frac{1}{H_2F_2} = \frac{P_2I}{P_1I} \times \frac{1}{P_2F_2}$$

or, $$\frac{1}{f} = \frac{1}{P_1I} \cdot \frac{P_2I}{P_2F_2} \quad \text{—} (6)$$

Now multiplying eqn (3) by P_2I, we get

$$\frac{P_2I}{P_2F_2} - \mu = \frac{1-\mu}{R_2} P_2I$$

or, $$\frac{P_2I}{P_2F_2} = \frac{1-\mu}{R_2} P_2I + \mu \quad \text{—} (7)$$

Then putting the eqn (7) in (6), we have

$$\frac{1}{f} = \frac{1}{P_1 I}\left[\frac{1-\mu}{\mu_2} P_2 I + \mu\right]$$

$$= \frac{1}{P_1 I}\left[\frac{1-\mu}{R_2}\left(P_1 I - P_1 P_2 + \mu\right)\right]$$

$$= \frac{1-\mu}{R_2} - \frac{1-\mu}{R_2}\cdot\frac{P_1 P_2}{P_1 I} + \frac{\mu}{P_1 I}$$

$$= \frac{1-\mu}{R_2} - \frac{1-\mu}{R_2}\cdot\frac{P_1 P_2}{P_1 I} + \frac{\mu}{P_1 I}$$

$$= \frac{-(\mu-1)}{R_2} + \frac{(\mu-1)t}{R_2}\cdot\frac{1}{P_1 I} + \frac{\mu}{P_1 I}$$

Putting the value of $\frac{1}{P_1 I}$ from (2) in the above we get

$$\frac{1}{f} = -\frac{(\mu-1)}{R_2} + \frac{(\mu-1)t}{R_2}\cdot\frac{(\mu-1)}{\mu R_1} + \frac{\mu(\mu-1)}{\mu R_1}$$

or, $$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu R_1 R_2}\right) \quad \text{--- (8)}$$

This is the required expression for focal length of the lens having thickness t.

→ In case of thin lens $t = 0$

$$\therefore \frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{—— (9)}$$

Which is thin lens formula.

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